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NOTE ON THE PRACTICAL APPLICATION OF A SUBSTITUTION GROUP IN SPHERICAL TRIGONOMETRY.

By DR. G. A. MILLER.

Several years ago Professor Study published a very valuable article* on spherical trigonometry in which he demonstrated the fertility of the group concept in this branch of elementary mathematics. His efforts were directed toward making the theory more general and more complete, and he explicitly states that the practical geometer would not find any results in his article which he could directly employ.

In what follows, our aim, on the contrary, is entirely practical. It is our belief that a fair knowledge of spherical trigonometry can be obtained more quickly by devoting one lesson to the substitution group given below than would otherwise be possible.

We let a, b, c represent the three sides of a spherical triangle, and α, β. γ the supplements of the angles opposite these sides, taken in order. From the polar triangle we have that the substitution

$$a\alpha.b\beta.c\gamma \dagger$$

will transform any formula into one which is equally true. It is also evident that any formula which has been derived without assigning any particular value to any one of the six parts is transformed into a formula which is equally true by means of the substitutions

$$ab.\alpha\beta \qquad bc.\beta\gamma.$$

Hence such a formula is transformed into one that is equally true if these substitutions are applied successively. In other words, any substitution of the group generated by the given three substitutions transforms any such formula into one that is equally true. This substitution group (G) is well known. It is composed of the following substitutions :

$$\begin{array}{rl}
1 & \\
abc.\alpha\beta\gamma & a\alpha.b\beta.c\gamma \\
acb.\alpha\gamma\beta & a\beta c\alpha b\gamma \\
ab.\alpha\beta & a\gamma b\alpha c\beta \\
ac.\alpha\gamma & a\beta.b\alpha.c\gamma \\
bc.\beta\gamma & a\gamma.b\beta.c\alpha \\
 & a\alpha.b\gamma.c\beta
\end{array}$$

If a general formula like

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*Study, *Saechsischen Abhandlungen*, 20, pp. 87—231.

†This means that a is replaced by α and that α is replaced by a, etc.

$$\sin a \cos \beta + \cos b \sin c + \sin b \cos c \cos \alpha = 0$$

is not transformed into itself by any one of the substitutions of G besides unity, we obtain 12 and only 12 formulae from it by transforming it by the substitutions of G. From each one of these 12 formulae we may obtain the other 11 by applying to it the substitutions of G which differ from unity. These 12 formulae thus form a closed system in regard to G. It is of interest to observe the closed systems in regard to the different subgroups of G and to interpret them geometrically. When a general formula like

$$\cos a = \cos b \cos c - \sin b \sin c \cos \alpha$$

is transformed into itself by one substitution of G which differs from unity $(bc.\beta\gamma)$ we may obtain 6 and only 6 formulae from it by transforming it by all the substitutions of G, and from each one of these 6 we may obtain the other 5 in a similar manner. Each of these six formulae may be obtained from any given one of them by transforming it by means of either one of two substitutions of G.

In general, if such a formula is transformed into itself by just k substitutions of G we may obtain $12 \div k$ formulae from it by transforming it by the substitutions of G, and all of these can be obtained from any one of them by transforming it by means of the substitutions of G, k substitutions giving the same formula in each case. It is clear that k must be a divisor of 12; *i. e.* the number of the substitutions of G that transform any general formula of the spherical triangle into itself is a divisor of 12.

If we replace α, β, γ by A, B, C respectively, the first half of the substitutions of G form a subgroup which may be employed in a similar manner to derive the formulae of the plane oblique triangle from each other. Hence this group might be made a very useful and general aid to remember the formulae of trigonometry. The explicit introduction of G into the text-books on spherical trigonometry would also furnish the means of making the student thoroughly acquainted with some of the elements of a concept of great fertility.

Cornell University, March, 1898.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

87. Proposed by E. W. MORRELL, A. M., Professor of Mathematics, Montpelier Seminary, Montpelier, Vt.

A and B set out from the same place, and in the same direction. A travels uniform-